

## Matrices: Identity Matrix, Inverse Matrix

### Identity Matrices:

$$(2 \times 2): I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (3 \times 3): I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4 \times 4): I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### Inverse Matrix:

$$(2 \times 2): A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Ex:

$$A = \begin{bmatrix} 1 & -3 \\ 5 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1(6) - (-3)(5)} \begin{bmatrix} 6 & 3 \\ -5 & 1 \end{bmatrix} = \frac{1}{6 + 15} \begin{bmatrix} 6 & 3 \\ -5 & 1 \end{bmatrix} = \frac{1}{21} \begin{bmatrix} 6 & 3 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} \frac{6}{21} & \frac{3}{21} \\ -\frac{5}{21} & \frac{1}{21} \end{bmatrix}$$

To CHECK:

$$\text{Rule: } A \times A^{-1} = I$$

$$\begin{aligned} A \times A^{-1} &= \begin{bmatrix} 1 & -3 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} \frac{6}{21} & \frac{3}{21} \\ -\frac{5}{21} & \frac{1}{21} \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 5 & 6 \end{bmatrix} \times \frac{1}{21} \begin{bmatrix} 6 & 3 \\ -5 & 1 \end{bmatrix} = \frac{1}{21} \begin{bmatrix} 1 & -3 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 6 & 3 \\ -5 & 1 \end{bmatrix} \\ &= \frac{1}{21} \begin{bmatrix} 1 & -3 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 6 & 3 \\ -5 & 1 \end{bmatrix} = \frac{1}{21} \begin{bmatrix} 1 \cdot 6 + (-3)(-5) & 1 \cdot 3 + (-3)(1) \\ 5 \cdot 6 + 6(-5) & 5 \cdot 3 + 6 \cdot 1 \end{bmatrix} = \frac{1}{21} \begin{bmatrix} 21 & 0 \\ 0 & 21 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$